but none were competitive" (p. 219). However, the co-operative element is couched in a competitive environment, such as a team relay competition. Similarly: "I'm not interested in how many people I beat, or what place I finish. It is me versus the clock" (p. 74). However, this vision is not consistent when an episode of playing the rules is explained. In the account, the protagonist constrains the possible solutions and repeatedly hands in answers (allowed by the rules) until she reaches the correct one.

As a teacher, I have always wrestled with the dominance of contest-oriented enrichment. This book, in spite of its strength of providing a valuable resource in math education, does not address non-competitive enrichment. Given the Canadian math landscape, I was not surprised by this. However, teachers should be aware that the book provides an in-depth view of a very specific form of enrichment.

Overall, Richard Hoshino has provided an important contribution to math educators and should be applauded for this book. The spirit it brings makes it an excellent enrichment prospect for high school students. It is well suited to school libraries, a classroom bookshelf, and as a potential motivational gift for students who deserve special treatment. ▲

▲ Thinking About Teaching, Learning, and Mathematical Mindsets Leads Me to Learning Skills

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"All learning begins when our comfortable ideas turn out to be inadequate." ~ John Dewey

The Ontario math curriculum states that assessment and instruction should be seamless (OME, 2010); others say that classroom management and instruction should be seamless (e.g., Jones, Jones, & Jones, 2000); and some state that curriculum and assessment should be seamless (e.g., Drake, Reid, & Kolohon, 2014). Each of these statements comes from a particular perspective that is theoretical, practical, or a combination of both. I would like to focus these perspectives into a mathematics classroom context—the space teachers live and work in for hours a day with hundreds of students. It is what has happened in my mathematics classroom that has helped me realize, over time, when my comfortable ideas of teaching mathematics became inadequate, when student learning was as uninspired as the routine of my teaching, and like a badly designed spiral curriculum, I was going around in circles and never really feeling like everything came together at the same time. Don't get me wrong, I saw regular successes in my students' achievement, and felt I had many good teaching moments, but a growing sense of inadequacy prodded me to broaden my knowledge base and then my professional practice.

I began to conceptualize my professional practice with the metaphor of the centre of gravity, or centroid, of a

triangle. Imagine a piece of cardboard, cut into the shape of a triangle, balanced on the end of a pencil placed at its centroid. Consider the vertices to represent classroom management, instruction, and assessment (see Figure 1), and this balanced triangle represents a teacher's professional practice. Knowing a single metaphor may not be able to capture everything, and for the sake of argument and simplification, I imagine that these three vertices represent the sum of all the wisdom and understanding of what teaching is all about—everything we know about teaching can be captured within the metaphor of this triangle. The challenge for the classroom teacher is to maintain that balance in professional practice—do not let the triangle get too heavy on any one side or vertex and slip off the pencil. However, this can be difficult because we continue to learn so much about the teaching and learning of mathematics as we develop knowledge and skills in these areas of professional practice, and we risk upsetting the balance if we focus too much on any one aspect. So, as I learn, I continuously reflect on how classroom management, assessment, and instruction are being affected, and how their interaction (rather than just the sum of their parts) must be considered when I implement any new learning in my classroom teaching. This is my perspective on the lived experience of the classroom teacher—that context matters, and this balance is achievable, and desirable.

Figure 1. *A centroid metaphor for classroom practice*

When I think of the students in my classes, I appreciate that they are more than just students of a classroom mathematics course. They are also citizens of our current society, and twenty-first-century skills are becoming an important outcome from schooling and formal education. An overview of these twenty-first-century skills identifies them as critical thinking, problem solving, reasoning, creativity, imagination, innovation, perseverance, self-direction, self-discipline, initiative, collaboration, leadership, and active multiple literacies. These skills are appearing to be valuable to my students' expressions of achievement and mathematical performance, but more importantly, these skills are becoming increasingly visible in the policies, documents, and curriculum for mathematics courses and schools in which I teach. The current vehicle to achieve something like facility in twenty-first-century skills is our schools and our classrooms. I wonder how a student can learn these twenty-first-century skills, as well as the mathematics of the curriculum, without the skills of learning, or Learning Skills. When I ask my students, they agree that it is hard to learn something without Learning Skills.

As teachers, we also develop twenty-first-century skills, such as our own multiple literacies, or professional literacies (Pyper, Chin, & Reeve, 2015). Current thinking about learning is giving us some new language about teaching and learning. Professional literacies of learning and teaching are expanding with ideas like "fixed" and "growth mindsets" (see Dweck, 2006). These ideas in particular represent concepts of learning and thinking that are providing insight into learners' motivations, efforts, and identities. These ideas and issues are important to teachers and students. We wrestle with them daily in our lessons, tasks, and classrooms.

Growth mindsets are important, and they develop habits of mind. "A habit of mind is a pattern of intellectual behaviours that leads to productive actions" (Costa & Kallick, 2008, p. 16). To explain the basic nature of this pattern of intellectual behaviours, Costa and Kallick describe six dimensions: i. value – choosing to employ one pattern of intellectual behaviours over others; ii. inclination – feeling a tendency to employ this pattern; iii. sensitivity – perceiving opportunities to employ this pattern; iv. capability – possessing the skills and capacities to carry through with this pattern; v. commitment – constantly reflecting on and improving performance; and vi. policy – making it a personal policy to embed this pattern into one's thinking and actions. Some of these dimensions sound strikingly similar to the Ontario curriculum Learning Skills; for example, inclination is like initiative, sensitivity points to organization, and commitment appears under self-regulation as persistence.

But what are the specific habits of mind? Costa and Kallick (2008) list 16 habits of mind (see Table 1). Of the 16 they identify, a number of them feel like they align with other constructs and ideas about learning, which lends support to the premise that these habits of mind are authentic to our experiences of learning. Table 1 contains a partial list of some of these other constructs I can see fitting in. As you read the list, what ideas and constructs of learning are you thinking about? (For further details

about these habits of mind, see Costa and Kallick's book.)

Habits of Mind	Connections
Persisting	→ Resilience, self-efficacy
Managing impulsivity	→ Self-regulation
Listening with understanding and empathy	
Thinking flexibly	→ Mathematical processes, problem solving
Thinking about thinking (metacognition)	
Striving for accuracy	
Questioning and posing problems	→ Assessment as learning
Applying past knowledge to new situations	→ "Thinking" Achievement Category
Thinking and communicating with clarity and precision	→ "Communication" Achievement Category
Gathering data through all senses	
Creating, imagining, innovating	
Responding with wonderment and awe	
Taking responsible risks	→ Learning Skill
Finding humour	
Thinking interdependently	
Remaining open to continuous learning	

Table 1. *The 16 Habits of Mind (Costa & Kallick, 2008) and some associated connections to other teaching and learning constructs*

It will be beneficial for one's learning to make these habits of mind automatic or natural states of thinking because "habituation (Mason & Davis, 1989) is a fundamental process through which people come to know" (Mason & Spence, 1999, p. 152). Knowing, now, is likely the crux of the issue. What do we know, and how do we know it? It may be nice and wonderful to develop and have habits of mind at our disposal, but what is the epistemological sense of the knowledge that occurs because of these habits? What is the nature of knowing that we are habitually acting upon?

Here are three ways of "knowing" from a mathematics perspective. Cuoco, Goldenberg, and Mark (2010) list eight mathematical habits of mind: pattern sniffing, experimenting, describing, tinkering, inventing, visualizing, conjecturing, and guessing. These habits of mind are described through a geometrical approach and an algebraic approach to thinking (see Cuoco, Goldenberg, & Mark, 1996 for details on these two approaches). In the process of exploring and explicating the assessment of mathematics, the Harvard School of Education (1995) described mathematical inquiry (or thinking) as five phases using eight verbs: hypothesising, modelling and formulating, transforming and manipulating, inferring and concluding, and communicating. These may look familiar to many who taught or experienced the 1999 version of the Grades 9 and 10 mathematics curriculum—these verbs formed a foundation to the pedagogical approach to that curriculum. Mason and Spence (1999) described two ways of knowing mathematics, knowing-about and knowing-to. In essence, knowing-about consists of knowing-that (the facts), knowing-how (the procedures), and knowing-why (the reasons). Knowing-to, though, is the ultimate goal, and this kind of knowing is all about knowing to act when presented with a problem, a situation, or question. (See Figure 2 for a visual graphic of "knowing.")

"Once the moment of knowing-to takes place, knowing-how takes over to exploit the fresh idea; knowing-that forms the ground, the base energy upon which all else depends and on which actions depend; knowing-why provides an overview and sense of direction that supports connection and modification if difficulties arise en route" (p. 146).

As they described these two ways of knowing, Mason & Spence linked them to assessment. The easy kind of assessment uses indicators of knowing-about, and the more challenging kind of assessment uses indicators of knowing-to. Their contention is that it is the more challenging kind of assessment and mathematical performance (knowing-to) that we, as teachers, need to emphasise for the benefit of our students' learning.

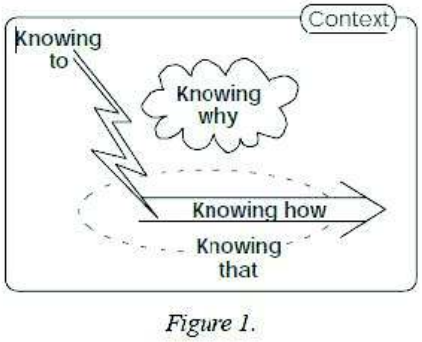

Figure 2. *Figure 1 from Mason & Spence (1999), page 145, illustrating the relationship between knowing-about and knowing-to*

		LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
Responsibility	*Coursework (at home and in class)*	Completes ½ the work, submits consistently late.	Completes ¾ of the work, submits occasionally late.	**Completes all work, submits on time every time.**	Completes all the work, submits on time and sometimes earlier.
	Choices	Avoids, ignores, resists ownership for choices.	Accepts responsibility for choices after it is brought to one's attention.	**Takes ownership and responsibility for choices.**	Takes ownership and proactively makes necessary choices.
Independent Work	*Accomplishing tasks*	Does not start work until the teacher makes a personal request, does not complete task.	Starts work after a couple of requests by the teacher, uses about half the class period effectively.	**Starts when told and uses the class period to complete task.**	Starts right away and uses the full class period to complete the task.
	Following instructions	Misses most instructions from the teacher, follows others' lead, copies work.	Misses some of the instructions from the teacher.	**Listens to instructions and can get to work.**	Listens to all instructions and provides leadership to others.
Initiative	*Academic risk taking*	Not willing to take risks, needs continuous prompting from others to try something new.	Hesitant, needs many prompts from others to try something new.	**Willing, interested, needs few prompts from others to try something new.**	Willing, curious, innovative, does not need prompts from anyone.
	Attitude	Focuses on own wants and needs. Consistently negative outlook on the course.	Focuses on own interests and "rights." Often pessimistic outlook on the course.	**Works for own and others' rights and responsibilities. Positive outlook on the course.**	Works and advocates for own and others' rights and responsibilities. Optimistic and upbeat.
Organization	*Prioritizing*	Makes no plans. Does not prioritize.	Makes a plan. Does things in the order they are written.	**Establishes a plan and prioritizes.**	Establishes, prioritizes, and periodically updates the plan.
	Managing resources	Comes to class with nothing.	Brings some things, remembers once he/she gets to the classroom.	**Before coming to class, identifies, gathers, then uses all resources.**	Proactively identifies, gathers, uses, and evaluates the use of all resources.
Collaboration	*Peer relations/ Working together*	Dependent. Doesn't participate, just sits there.	Independent. Doesn't share with group.	**Interdependent. Shares ideas, encourages others' participation.**	Effective, positive, interdependence. Encourages critical thinking, elaborates on anyone's ideas. Everyone feels valued.
Self-Regulation	*Goals*	Has no individual goals.	Sets unrealistic or vague goals.	**Sets specific and achievable goals.**	Sets, monitors, and re-evaluates measurable goals.
	Persistence	First, and often the only, reaction is to give up.	First attempt is considered acceptable, works for a "pass."	**Sticks to it and doesn't give up.**	Will improve work, even after it has been assessed.

Figure 3. *Learning Skills & Work Habits (2010)*

To consider the growth and development of knowing-to, what might we need to attend, to help our students develop these habits of mind? Costa and Kallick (2008) suggest we explore meanings, expand capacities, extend values, build commitment, and increase alertness. Again, I notice an alignment of these four goals with the Ontario curricular Learning Skills and mathematics processes. I feel embedding Learning Skills into our daily classroom learning will meet each of these goals, which leads me to the questions, how would one teach *for* the habits of mind, how would one teach of the habits of mind, and how would one teach *with* the habits of mind?

The centroid and triangle metaphor is my framework for teaching—thinking of the seamlessness of teaching/instruction, assessment *for* and *of* learning, and classroom management of students' learning, and considering mathematical mindsets in the context of a math classroom. So that teaching turns into students' learning, the potential lynchpin to all of this is Learning Skills. Learning Skills are described in *Growing Success*, initially in the 1999 version, and now in the revised 2010 document. As a Grade 9 math teacher back in 1999, I appreciated the value of Learning Skills, but found it difficult to imagine how to incorporate them into my classroom practice. During a staff meeting before report card time in 2000, some colleagues presented a way to capture Learning Skills in a one-off opportunity just in time to gather some evidence and be able to add something to the report card under Learning Skills. I remember listening to their presentation and thinking that this is much too small a sample of Learning Skills evidence for me to accurately describe and evaluate my students' Learning Skill levels for a report card. Walking away from that presentation, I thought of the tracking sheet I was using for monitoring aspects of student classroom learning (see Pyper, 2005 for details about the tracking sheet). This was a somewhat teacher-centric and teacher-managed process, but wondering if I could add Learning Skills to the tracking sheet inspired me to imagine a more student-centric and student-managed process. I developed a Learning Skills rubric from the then four Learning Skills of the curriculum. I used the official document's titles and descriptions and transformed the rubric criteria into language students would accept and understand. I then wrote the four levels of quality of the criteria, again in language students would understand. I implemented and tested in my classrooms, and responding to students questions and input over the next two semesters, improved the wording in the rubric. Over the years, this was used by members of the math department and the summer school math teachers. When I moved to a new school in 2009, three other teachers in the new school (teaching English, History, and Science) started using this Learning Skills rubric and tracking sheet. When the new *Growing Success* document was published with six Learning Skills and included modifications to the descriptions, the four of us tweaked the Learning Skills rubric for the 2010 school year. This is the rubric you see in Figure 3.

Figure 4. *Learning Skills rubric on a classroom wall*

There are a number of features to the design and implementation of this Learning Skills rubric. Size is important—it needs to be large and wall mounted for everyone to see easily. I use regular 8.5x11 sheets of paper in plastic page protectors for each cell of the rubric. Colour is important—the Level 3 column is the "standard," as defined by the Ontario curriculum, so I wanted a colour that students' eyes would be drawn to first. Level 4 is a goal, not a standard, and so I wanted students to feel purposeful if they were trying to achieve a Level 4, hence the darker colour red. This is also a colour that requires one to sometimes go up to the wall itself to read the words—in a small sense, the physical demand of going to the wall increases the motivation to know what it says and then act accordingly. The rubric's Levels 1 and 2 columns are the colour of the wall, designed to blend in and not look appealing—I don't want students to select Level 1 or 2 as their goal, so I want these pages to either be missed or ignored. The criteria for each Learning Skill

is chosen to reflect the behaviour or learning skill I have seen most often in need and which might benefit students' learning most. The descriptors in each level are purposefully worded to make Levels 1 and 2 unappealing to students, so they strive for Level 3 as a standard level of achievement and behaviour. There is a tension between an official pass being 50% or Level 1, and the standard level of achievement being 75% or Level 3. I have chosen to emphasise the standard level of achievement as a beneficial focus for students' success. See Figure 4 for an example of what this rubric could look like on a classroom wall.

For the first six weeks of class, I emphasize a Learning Skill each week. Starting from the top of the rubric, and each week, moving down to the bottom, we discuss the Learning Skill and focus on Level 3, self-assess with it, and learn what it means to embed Learning Skills in a metacognitive way while learning math. For the next two months, I select a different Learning Skill each week to align with what I have planned pedagogically. Soon students are selecting the Learning Skill for the week, and then by the last third of the course, we pick Learning Skills as necessary for individual days or periods of time. For the first six weeks, we are assessing Learning Skills together frequently, but by the middle of the course, we are assessing Learning Skills about three times a week.

I have seen students experience great success with this Learning Skills rubric. Students start to use the language of the Learning Skills (think learning skills literacy) and apply it to themselves and their peers (think assessment for and as learning). For example, at the end of one class and during the time needed for the class to self-assess and write on their tracking sheets, one student spoke out loud in class to say that another student (and named the student) should get a Level 4 for initiative because that student had been going around and helping people. The value of this story is that both were at-risk students, one much older in age, and both had not expressed much value for school in their previous years as a student. (The speaker, unfortunately, dropped out a month later, but the recipient of the praise stayed, finished this second attempt at the Essentials 10 course, and then completed the Applied 9 and 10 courses the next school year. In a personal communication at the end of the Grade 10 Applied course exam, the student attributed this to the use of Learning Skills in his classes—as soon as he started to figure out how to learn, the math got easier and easier.)

The implementation of this Learning Skills rubric is successful for me in my math classroom because it aligns with and supports so many aspects of instruction, assessment, and classroom management (think of my triangle metaphor.) This Learning Skills rubric is also successful for students in my math classroom because of its alignment with, and support for developing twenty-first-century skills, growth mindsets, habits of mind, and mathematical mindsets. Students feel successful in their learning because of the continuous application, its authentic and natural integration into mathematics lessons, and the ease with which students find that their improvement of the skills of learning invariably improve their achievement. They see this improvement in increased attendance, greater content learning and retention, and improved course marks. Student ownership over their learning becomes natural, and this is a most wonderful feeling as a teacher.

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