▲ E-Kwatee-Ons – From Mind Games to Algebra Part 2

JAMIE PYPER
E-MAIL: pyperj@queensu.ca
WEBSITE:
www.educ.queensu.ca/research/collaborations/mste.html

Jamie is an Assistant Professor of Mathematics Education, and the Coordinator of a research and curriculum development team, the Mathematics, Science, Technology Education Group (MSTE), at the Faculty of Education, Queen's University, he spent 20 years teaching high school mathematics in two school boards in Ontario and mathematics education at Western University. He has been a member of OAME since 1995 and is often a workshop presenter at OAME and local chapter conferences.

I often teach the Grade 9 or 10 students who struggle with math. An often-seen treatment of solving equations starts with equations that require only one transformation: $ax = c$, or $\frac{x}{b}$, or $x + b = c$. Students almost always exclaim, "These are so easy! We can do them in our heads. Why do we have to write all these steps?" Brilliantly, the Ontario Ministry of Education (OME) (2005b) Grades 9 and 10 curriculum and the NCTM Standards (2000) support and encourage a mental ability. For example, "[t]hey should become fluent in performing such manipulations by appropriate *means—mentally*, by hand, or by machine—to solve equations …" (NCTM, 2000, p. 296, italic added), and

> Mental computation involves calculations done in the mind, with little or no use of paper and pencil. Students who have developed the ability to calculate mentally can select from and use a variety of procedures that take advantage of their knowledge and understanding of numbers, the operations, and their properties (OME, 2005b, p. 15).

However, emphasizing the traditional paper-and-pencil approach of solving single-step equations first, seems to lose students' interest. By the time students get to equations that could use a good algebraic, multi-step solution process, students are dismissive of equations and do not care to learn a formal algebraic method. Thinking from a differentiated instruction perspective, I decided to let students solve those one-step equations in their head, and instead, focus my teaching energies on a process that would solve (almost) any equation that involved two or more operations, such as $ax + b = c$, or $\frac{x}{b} + c = d$, or, $\frac{x+b}{c} + d = e$, the equations that are often more difficult for students to solve mentally just by looking at them.

What follows is a unit I use for students' learning of solving equations that considers students' attention spans, attention to details, and learning styles. The unit may run from three to six days, depending upon my students' learning needs and the order I have planned for curriculum topics in the course I am teaching. Time is relatively flexible because of an overriding belief I have-that if I do not take the time now, then I will reteach "solving equations" in every unit for the rest of the course. Students have experienced learning opportunities for solving equations (e.g., OME (2005a) curriculum by Grade 8), but often do not recognize they can "solve an equation" for the variable in different mathematical contexts or topics. This unit provides an engaging discovery atmosphere, where students generate a personally memorable algorithm to solve for unknown values.

I start with a mental game of number manipulation that all students can perform in their heads and feel comfortable speaking out loud. Then, I gradually incorporate a paper format to the game, which transforms over the lessons into a formal algebraic and algorithmic process of solving equations with which we are all familiar.

This is the unit plan:

Day 1 E-kwatee-ons game in our heads.
Day 2 E-kwatee-ons game board on paper.
Day 3 E-kwatee-ons and writing each step as an arithmetic operation.
Day 4 Resolving E-kwatee-ons into the formal solving equations algorithm on paper.
Day … Practice and extensions into more complicated equations as needed.
Day n Practice, Equations Bingo, evaluation.

Day 1

This whole first lesson is verbal and conversational with students in the class as a whole group and with students in pairs. Whether it is learning styles or students' perceptions of how they learn best, Day 1 appeals to many students because of its verbal and conversational emphasis. It can be challenging to incorporate all learning styles simultaneously, and I find an emphasis on one learning style on one day, and an emphasis on another learning style on another day, often allows students to engage when an activity appeals to a strength in a particular learning style, and is then supported by peers and teachers when students work on an activity that emphasizes a learning style in which they are weaker. Details of the verbal lesson process of the game appear in the *Gazette 51*(2), 29–31 for **E-kwatee-ons – From Mind Games to Algebra.**

Day 2

Like any really good game, there is a game board. For example, think of Tic Tac Toe, Sorry, and Monopoly. E-kwatee-ons has a game board too. I have students write the game board in their notes (see Figure 1 for the lesson "note" I put on the chalkboard). I then use this game board with an example of a game we played in our heads the previous day.

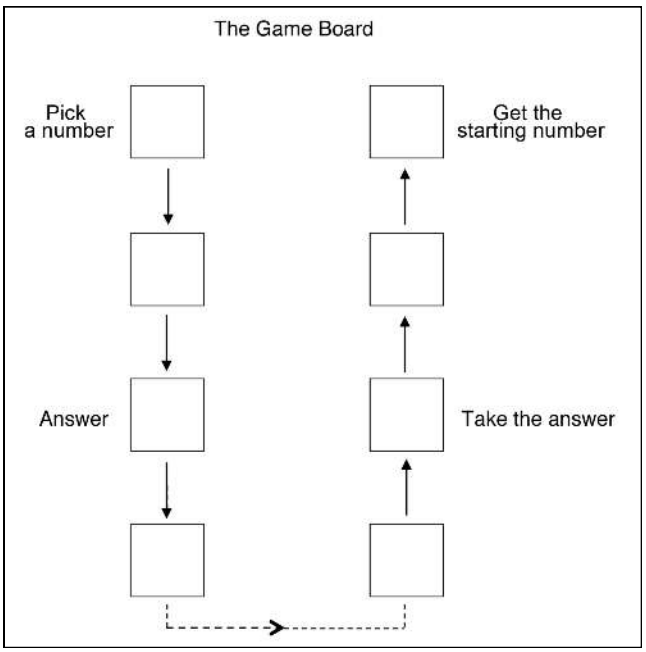

Fig. 1. *The Game Board*

Students play the game with the game board in pairs, again in a pattern of increasing size of numbers and number of operations (see Figure 2 for how the lesson note on the chalkboard develops for this day).

Playing the Game
"Pick any number between 1 and 9." "Multiply that number by 2 – keep your answer in your head." "Add 3 to that number – keep your answer in your head."
"What is the value you are thinking of right now?"
A possible answer is 15.

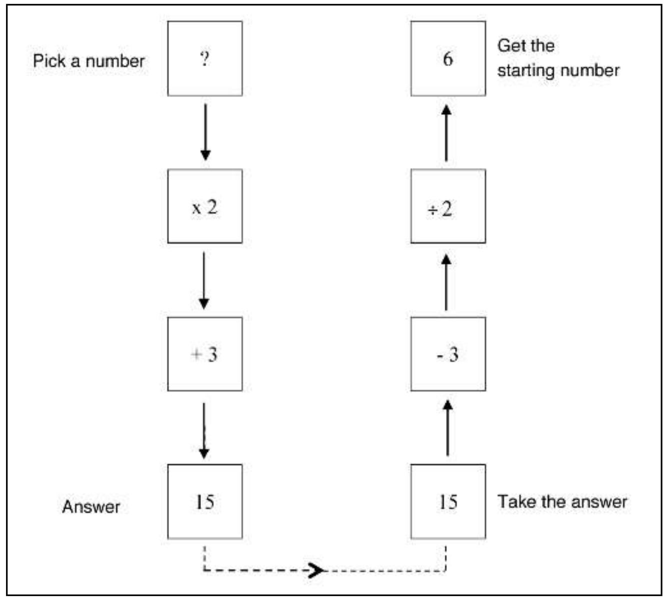

Fig. 2. *Playing the Game*

Day 3

On the third day of the unit, I suggest there might be another way to write down the game, and using a game example from a previous day, I say the game and write each step on the board. Figure 3 illustrates the step-by-step nature of this development.

Words I would speak to the class	What I would write on the board
"Pick any number between one and nine."	?
"Multiply that by 2."	2 x ?
"Add 3 to that."	2 x ? + 3
"What is your answer? Brent? 11"	2 x ? + 3 *is* 11
"Now, what is another way to write 'is'?"	2 x ? + 3= 11
"And what do we usually write instead of a question mark when we don't know the value?"	2 x x + 3= 11
"And now remember, the multiplication sign often doesn't get written between a number and x, so it would look like this"	2x + 3= 11

Fig. 3. *Writing the spoken game on paper.*

Figure 4 is the lesson note I put on the chalkboard. Using the acronym "BEDMAS," we work through the note, do some examples, and homework comes from the textbook-questions from a chapter on solving equations. An option I have used is to have students make up their own equations, solve them, and quiz the person beside them, or do this at home and have the person at home solve the equation, using the game. This has the advantage of providing an opportunity for the students to teach this skill and the game-an excellent assessment as learning (Ontario Ministry of Education, 2010) strategy.

Playing the game with a textbook

Solving means to look for a value for the variable that makes the equation true.

Use BEDMAS to help you write the boxes for the game. Then go backwards through the boxes to find x.

Example: Solve $5x - 14 = 16$

Solution:

x → $\times 5$ → -14 → 16

$16 + 14 = 30$

$30 \div 5 = 6$

$6 = x$

Check: L.S. $= 5x - 14$

$= 5(6) - 14$

$= 30 - 14$

$= 16$

$=$ *R.S.*

Example: Solve $3(x + 4) = 33$

Solution:

x → $+4$ → $\times 3$ → 33

$33 \div 3 = 11$

$11 - 4 = 7$

$7 = x$

Check: L.S. $= 3(x + 4)$

$= 3(7) + 4$

$= 3(11)$

$= 33$

$=$ *R.S.*

Fig. 4. *Playing the game with a textbook*

Day 4 and onwards.

The next phase of the unit moves students into a more formal and algebraic method of solving equations. The algorithm I tend to use is the "balance" method of performing the same operation on both sides of the equation in an effort to isolate the variable. Figure 5 is the chalkboard note for this lesson. The left side of the note illustrates the game solution to solving an equation, and the right side of the note illustrates a more formal algebraic method. Once both methods are written on the chalkboard, we explore and emphasize the similarity of the steps, operations, and results of the game and the formal algebraic method.

Solving Equations More Formally

Use BEDMAS to help you think or write the boxes for the game.

Then use the inverse operations as you go backwards through the boxes to find *x*.

Example: Solve $\frac{x-4}{3} = 8$

Game Solution: $\frac{x-4}{3} = 8$

x → -4 → $\div 3$ → 8

$8 \times 3 = 24$

$28 = x$

More formal method:

$$\frac{x-4}{3} = 8$$

$$3\left(\frac{x-4}{3}\right) = 3(8)$$

$$(x-4) = 24$$

$$x - 4 + 4 = 24 + 4$$

$$x = 28$$

Fig. 5. *Solving equations more formally*

For some students, this has been a transformative experience. Some students move quickly to the formal algebraic method, while others continue to use the modified game method of Figure 4. There are a few students who cannot grasp any part of the algebraic method and solve all their equations with the game board of Figure 1. I accept any of the three methods for solving equations on the final exam. Each method shows mathematical thinking and mathematical communication appropriate for the learner's stage of development. I find that students who use the modified game method of solving equations for the duration of this course often leave it behind and solve equations naturally with the more formal algebraic method once they start their next math course.

Solving Equations When You Cannot Play the Game at First

Step 1 Simplify each side.
Step 2 Move variables to one side.
Step 3 Find the value. (Repeat steps, if necessary.)

Example: Solve $5x - 10 = 3x - 3x + 6$ ("collecting variables to one side)

Solution: $5x - 10 = 3x + 6$

$5x - 3x - 10 = 3x - 3x + 6$

Remember: Keep the equation balanced do the same thing on both sides of the equation.

$2x - 10 = 6$ (play the game, or solve more formally)

...

Fig. 6. *Solving more complicated equations*

There are many developments to pedagogical conceptions and related teaching and learning classroom practice, such as constructivism and differentiated instruction. From a constructivist perspective, teaching strategies in our classrooms should provide students with opportunities to connect prior knowledge and experiences to current knowledge and experiences. Pelech (2010) suggests a "bridging question strategy" to connect "everyday, manifest knowledge to an academic concept the students are learning" (p. 121). The E-kwatee-ons game is the bridging strategy between the prior knowledge and experiences of thinking in our heads of operations with numbers and games and magic, and the academic concept of solving equations algebraically.

From a differentiated instruction perspective, the E-kwatee-ons game provides an "appropriate challenge" to attain the "essential understandings" with a continuous "evidence base" (Hume, 2008, p. 9). The *evidence base* appears naturally on paper because it is generally understood that games with a defined game board have a set of rules that must be followed. Otherwise, one is "cheating" and is called out loudly and immediately by fellow players. There are three distinct phases to using the game board of E-kwatee-ons on paper to solve equations: mental arithmetic and the game board (Figure 2), then the modified game board (Figure 4), then a more formal algebraic method. This strategy provides students with multiple representations of the same skill, and students have a choice of solution method, another fundamental element of differentiated instruction stated in the Ontario mathematics curriculum (OME, 2010), and NCTM's Standards (2000).

The *essential understanding* is the use, and emphasis, of the order of operations. It is used to simplify numerical expressions, and now, it can be seen to be used with algebraic expressions and solving equations. The nature of the *appropriate challenge* follows from the three phases of using the game board. Students learn one step at a time, with lots of practice (Jones, 1987). As students gain facility and confidence with solving equations, they develop paper-and-pencil-solving strategies to answer the question of "What do I do next?" When a student shows he or she is ready to take the next step, that is, to use a more sophisticated manner of solving equations, another more algebraic solution method is available (for example, see Figure 6).

The Ontario (2005b) curriculum's mathematical processes and NCTM's (2000) Connections standard suggest there is value in making connections among various mathematics topics and investigating the interplay of these topics. The ability to solve simple equations is a skill that transfers into the learning and investigating of other mathematics concepts such as similar triangles, angle measures in 2-D geometric figures, optimizing surface area and volumes, and develops into more complex solving processes and algorithms. It may be the success felt with initial concepts such as solving simple linear equations that offers possible opportunities for students to persevere and continue to experience and enjoy mathematics.

References

Hume, K. (2008). Start where they are. *Differentiating for success with the young adolescent*. Toronto: Pearson Professional Learning.

Jones, F.H. (1987). *Positive classroom instruction.* New York: McGraw-Hill.

NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Ontario Ministry of Education. (2005a). *The Ontario curriculum, grades 1–8: Mathematics, (revised).* Toronto: Queen's Printer for Ontario.

Ontario Ministry of Education. (2005b). *The Ontario curriculum, grades 9 and 10: Mathematics, (revised).* Toronto: Queen's Printer for Ontario.

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation, and reporting in Ontario schools.* Toronto: Queen's Printer for Ontario.

Pelech, J. (2010). *The comprehensive handbook of constructivist teaching. From theory to practice.* Charlotte, NC: Information Age Publishing. ▲